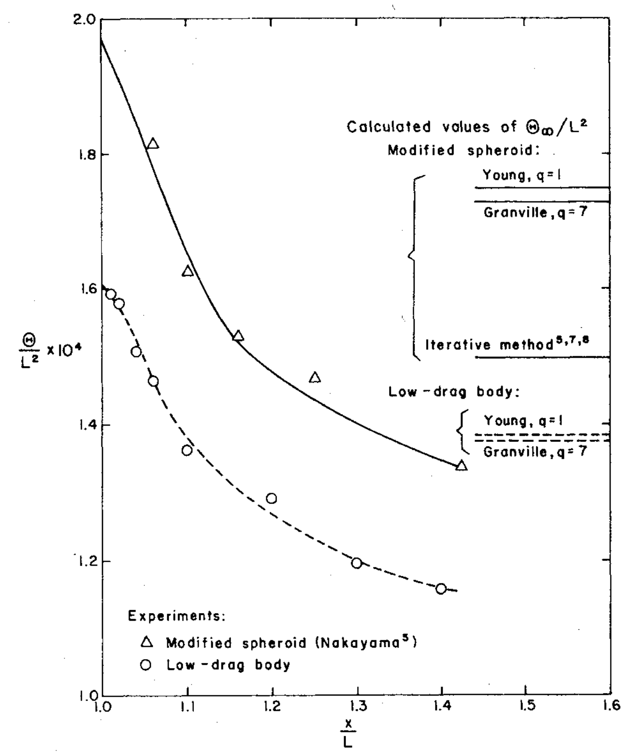

Fig. 1 Variation of Θ in the near wake of two bodies of revolution and calculated values of Θ_∞.

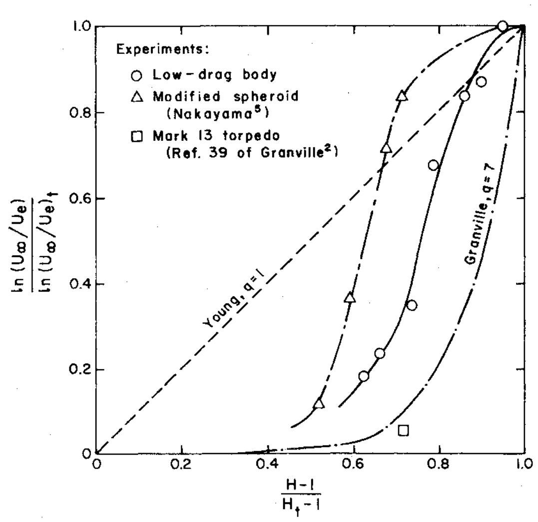

Fig. 2 Relation between U_∞/U_e and H in the wakes of bodies of revolution.

an overestimate of the drag coefficient when the formula of either Young or Granville is used. On the other hand, the prediction (Fig. 1) by an iterative method[5,7,8] which takes the term I_p into account is much closer to experiment.

Conclusions

It is of interest to observe that detailed experimental data in the near wake of bodies of revolution were not available at the time Young and Granville proposed their formulas. The relative success enjoyed by these formulas over the years may be attributed largely to the fact that their application was restricted to bodies whose tail configurations were such that the boundary layer either remained thin up to the tail or separated in the actual experiments before it became thick. In either case the pressure variation would not be large enough to avoided by proper design of the tail and the boundary layer becomes thick, as in the case of the low-drag "dolphin" bodies of current interest,[6] the older drag-prediction formulas cannot be relied upon to obtain satisfactory accuracy.

As has been pointed out in earlier studies,[3,4] the variation of static pressure across the boundary layer and the near wake implies a strong interaction between the boundary layer, the wake, and the external inviscid flow. This implies that conventional boundary-layer calculation methods cannot be relied upon to predict even the boundary-layer characteristics at the tail. A drag-prediction method which accounts for the flow interaction referred to has been developed in Refs. 5, 7, and 8.

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Water Fog Generation System for Subsonic Flow Visualization

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Introduction

THE problems of flow visualization in a gas have in the past been related to the undesirable qualities of the injected marking smoke.[1,2] The toxicity and corrosiveness of titanium tetrachloride are well-known to anyone who has used it.[3] The vaporization of "Type 1964 Fog Juice" or the burning of a wide variety of combustibles for the purpose of producing smoke have inherent cleanliness and safety problems.[4,5] Not only is a dense smoke usually acrid and harmful to breathe, but the methods of vaporization are dangerous from the standpoint of explosion and/or fire.[4] The

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obvious need for a unique system,[6] which would be clean, nontoxic, noncorrosive, nonflammable, and chemically inert in normal usage, has led to the development of a liquid nitrogen-stream fog generating system at the M.I.T. Aerophysics Laboratory.

Experimental Apparatus

The fog generator is basically a very simple device, as shown in Fig. 1, which allows steam and liquid nitrogen to be combined turbulently in a mixing nozzle, and then expelled. Liquid nitrogen is pumped from a Dewar through a control valve and into the midsection of the mixing nozzle. Compressed air is valved and pumped through a closed Erlenmeyer flask, containing hot or boiling water, where the air becomes saturated with water vapor at the flask temperature. It then flows into the end of the mixing nozzle, where it is cooled rapidly by mixture with liquid nitrogen to create a dense white fog, which is expelled from the nozzle (Fig. 2). In addition to the primary plumbing just described, a temperature feedback loop is employed to regulate the temperature of the mixing nozzle, which therefore loosely controls the temperature of the resultant fog. The feedback system senses the temperature of the nozzle through a five-junction thermopile, which is attached to a Honeywell Pyr-O-Vane temperature-control unit. The control unit opens and closes a resistive heating circuit, which regulates the temperature of the mixing nozzle and liquid nitrogen inlet tube. This configuration makes it possible to control the exit velocity of the fog, the fog optical density, and the fog temperature independently. To a first approximation, the operation is as follows:

Effluent fog volume flow (velocity) is controlled by adjusting the air flow into the boiler. Opacity is controlled by varying the boiler temperature (i.e., water flow rate). The fog temperature, and consequently the mass density, then is controlled by adjusting the flow rate of liquid nitrogen.

The presence of three interacting independent variables, however, makes adjustment sensitive. In order to prevent the fog from falling or rising as it travels through the stilling section, it is necessary to maintain the fog mass density very close to the inlet air density. This requires more exact control than would be required merely to prevent ice formation in the mixing nozzle. It appears that it may be necessary to hold the temperature of the fog within 1 or 2°F of the temperature for neutral buoyancy in order to prevent the thermal convection from generating vorticity in the injected fog stream.

Operational Experience with the Prototype System

The prototype mixing nozzle was designed to be mounted inside the contraction section of the M.I.T. Aerophysics Laboratory subsonic wind tunnel[7]; hence, its streamlined shape and small size (Fig. 2). This was not done, however, because it was difficult to fit all of the feed lines into the contraction section.

In its present configuration, the nozzle is mounted outside of the tunnel, and the fog is piped to the inlet of the contraction section with 1-in. i.d. plastic tubing (Fig. 3). This allows the fog generator cart with transformer and temperature control to be located where it will least affect the air moving into the inlet of the tunnel. The configuration also makes the system reasonably compact, portable, and easy to adjust to different needs. By use of the plastic tubing, fog has been piped several feet with little recondensation of the vapor. This makes it possible to locate the fog generator away from the injection point in the wind tunnel, but, more importantly, it permits the use of injection rakes, flow straighteners, and so forth, to provide a low-turbulence fog stream tube of the proper inlet diameter.

The optical appearance of the fog is almost as opaque as titanium tetrachloride smoke, and is comparable to oil or tobacco smoke. The reflectivity for photographic and viewing purposes is very much the same as the three kinds of smoke just mentioned. Its peristence downstream in the wind tunnel

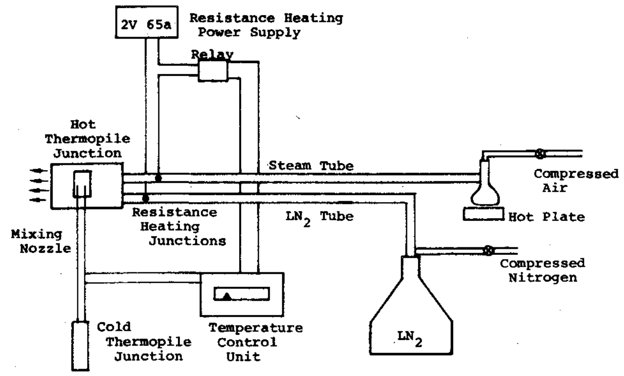

Fig. 1 Fog generator prototype configuration.

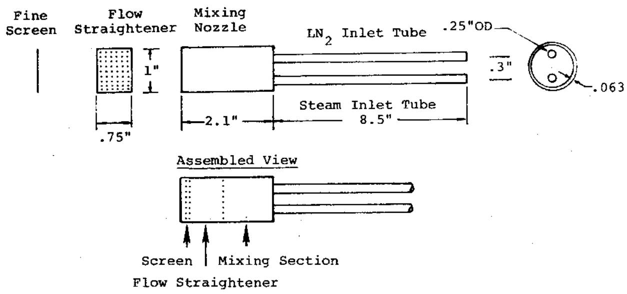

Fig. 2 Prototype mixing nozzle.

is different, however. Depending on the fog stream tube size, the level of turbulent mixing (dictated mainly by the tunnel speed), and the relative humidity of the surrounding air, the fog persists a given distance down the tunnel before it vaporizes into the air stream. This distance usually is greater than 15 ft, even at test section speeds of 125 fps. The useful length of the fog is about at the end of the test section in Fig. 3. The range of flow rates within the present system's operating capability is approximately 0–8 cfm of fog, requiring flow rates of LN_2 up to ½ ℓ/min. Unfortunately, more detailed measurements are not available at present. Figures 4 and 5 show visualization photographs of a magnetically suspended ring airfoil, which uses the system.

The present prototype fog generator has demonstrated the capability of producing optically dense water fog in the M.I.T. Aerophysics Laboratory subsonic wind tunnel. The technique appears well-suited for wind-tunnel flow visualization of all types. Experience with the prototype system has indicated several points that should be stressed in building a fog system, in addition to matching fog and wind stream velocities.

1) Pressure regulators and metering valves for liquid nitrogen and airflow control must be precise and stable.

2) Temperature control is required at two points, the mixing nozzle and boiler. This temperature control should be precise—perhaps ± 1°F with respect to the set temperature. The boiler should be regulated with respect to a set temperature (the fog opacity control). The effluent fog temperature should be controlled automatically to within a set difference from the wind-tunnel static temperature (buoyancy control). In addition, the mixing nozzle must be maintained above the freezing point.

3) Liquid nitrogen lines should be of small diameter and well-insulated to prevent boiling and slug flow in the lines which cause unsteadiness in the fog flow.

4) After generation, the fog can be straightened and smoothed with conventional techniques, but only plastic tubing should be used, since excessive condensation takes place on metal surfaces.

5) Fog tubes should be provided with drain holes, so that condensed fog can drip out.

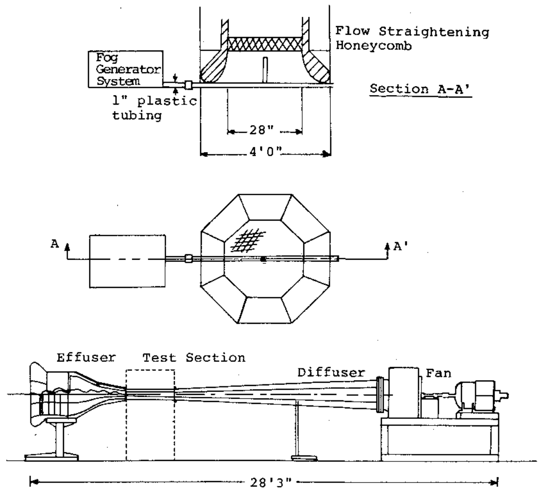

Fig. 3 Fog generator and subsonic wind tunnel.

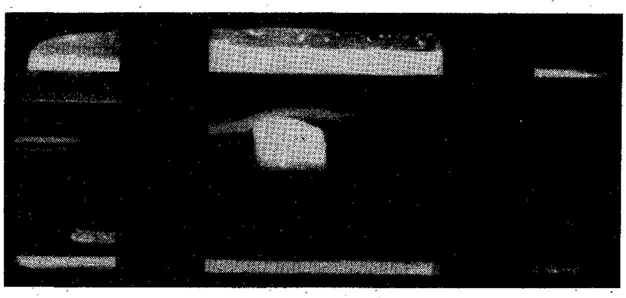

Fig. 4 Fog visualization photograph of magnetically suspended ring airfoil with separated flow at 85 fps (not spinning).

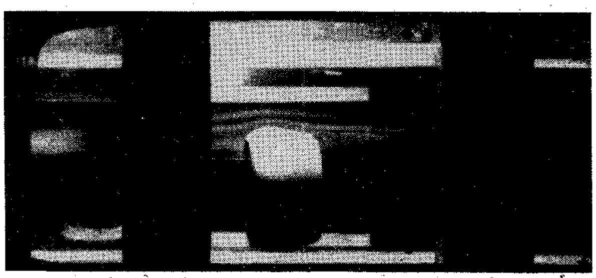

Fig. 5 Fog visualization photgraph of magnetically suspended ring airfoil with attached flow at 250 fps (not spinning).

The authors believe the water fog visualization technique also should be usable in atmospheric supersonic wind tunnels, if the total water flow rate is held sufficiently low to maintain the dewpoint in recirculation wind tunnels.

As suggested by a reviewer, other methods,[6] using higher vapor pressure substances, might be more useful for oscillatory, transient, or decelerating flows. It might be possible to use the present technique of liquid nitrogen mixing to quench the vapor phase of these other substances rapidly to produce a finer and denser smoke.

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Computer Algorithms for Computation of Kinematical Relations for Three Attitude Angle Systems

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Introduction

IN the study of attitude dynamics of spacecraft, it is commonplace to describe spacecraft attitude or orientation by a set of three attitude angles (typically Euler angles) through which a sequence of successive planar rotations is specified. In this description, a planar rotation is represented by a 3×3 orthogonal matrix, called a transformation matrix.[1] A general description of the orientation is generated by a single transformation matrix constructed as a matrix product of such matrices. In digital simulation of attitude dynamics,[2,3] a transformation matrix is often used together with another set of kinematical relations between angular velocities and the time derivatives of attitude angles (sometimes called Euler's kinematical equations). If a sequence of rotations is selected among many†, then the transformation matrix and angular velocity relations are uniquely determined.

Although the algebraic properties of such rotations have been studied extensively,[1,4] these kinematical relations are usually formed by manual calculation for a given ordered sequence and then programed on a digital computer. This is a laborious job especially when more than one coordinate systems must be employed.[2]

In this paper, a unified algorithm is developed for computation of the general transformation matrix on a digital computer ("general" in that if a sequence of any order is given, the transformation matrix is automatically generated without any manual calculation). An algorithm is also established for the general kinematical relations between angular velocities and the time derivatives of three attitude angles. In addition to these, algorithms for the inverse transformations are also developed, and thus a complete set of the kinematical relations is established for use in digital computer simulations. Particular emphasis is placed on the avoidance of unnecessary multiplications which otherwise cause excessive computation time.

Transformation Matrix

Consider the following coordinate systems; the first one (e.g., reference frame) is defined by the dextral (right-handed) orthogonal set of unit vectors (called a vector basis hereafter), $\{\boldsymbol{a}\}$, and the second one (e.g., body frame) by $\{\boldsymbol{d}\}$, and in addition to these, the two intermediate coordinate systems $\{\boldsymbol{b}\}$ and $\{\boldsymbol{c}\}$ which take place during a sequence of planar rotations.

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†A simple combinatory calculation shows that there exist twelve meaningful sequences, four of which are essentially different from each other.